Fluid Mechanics
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Lecture - 13
Fluid Statics Applications: Example Problems

Welcome all of you to Fluid Mechanics course. Today we are going to solve the problems on fluid statics.

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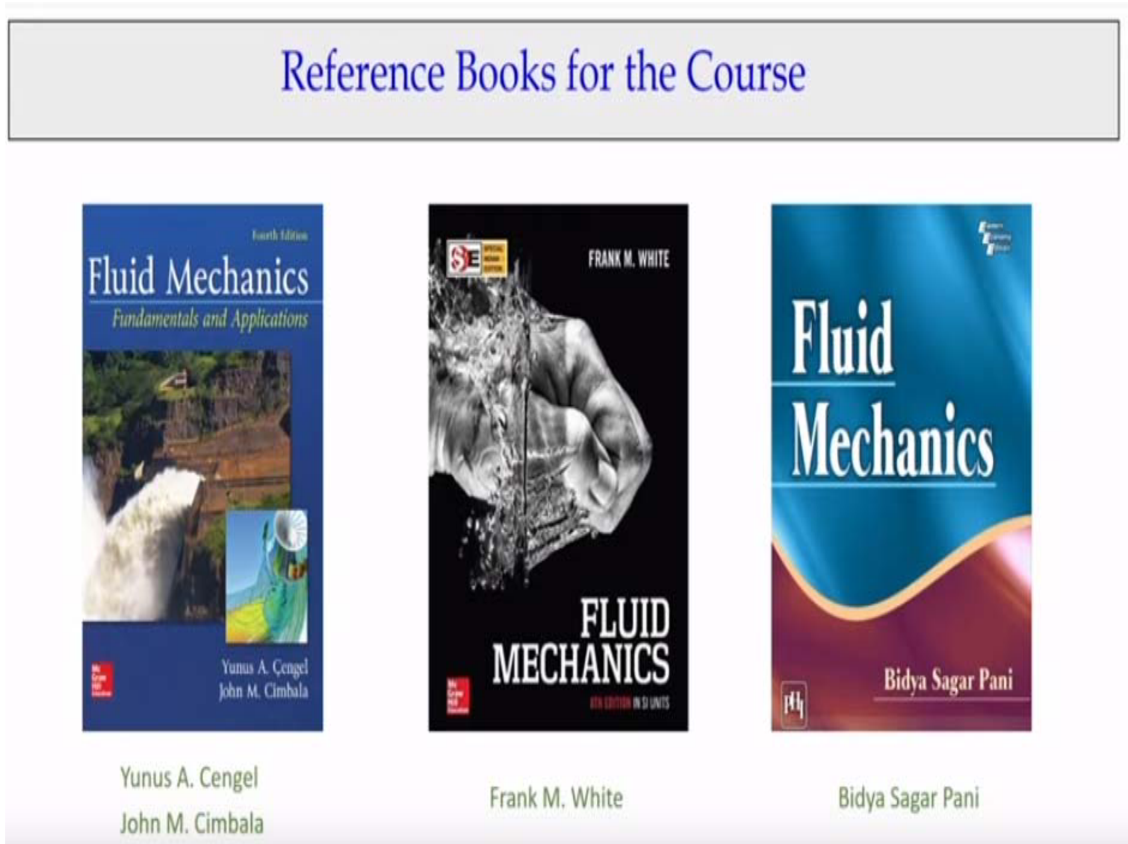

Before starting that again I repeat that these are the three books we follow to solve fluid mechanics problems. But today I will focus on the fluid mechanics problems in GATE exam and Engineering Service Exam.

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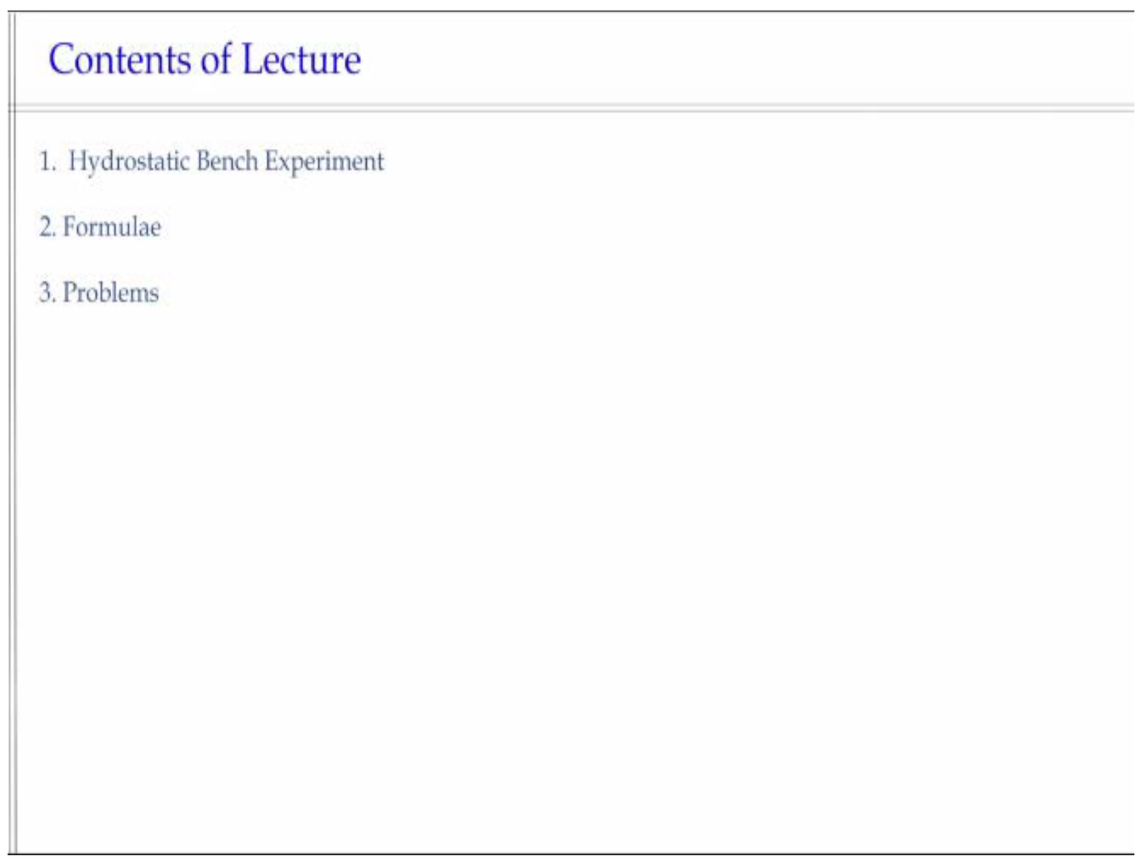

Looking that today I will cover with a introductions to hydrostatic bench experiment. Then some formulaes, then we will solve 10 problems from GATE exam and Engineering Service Exam.

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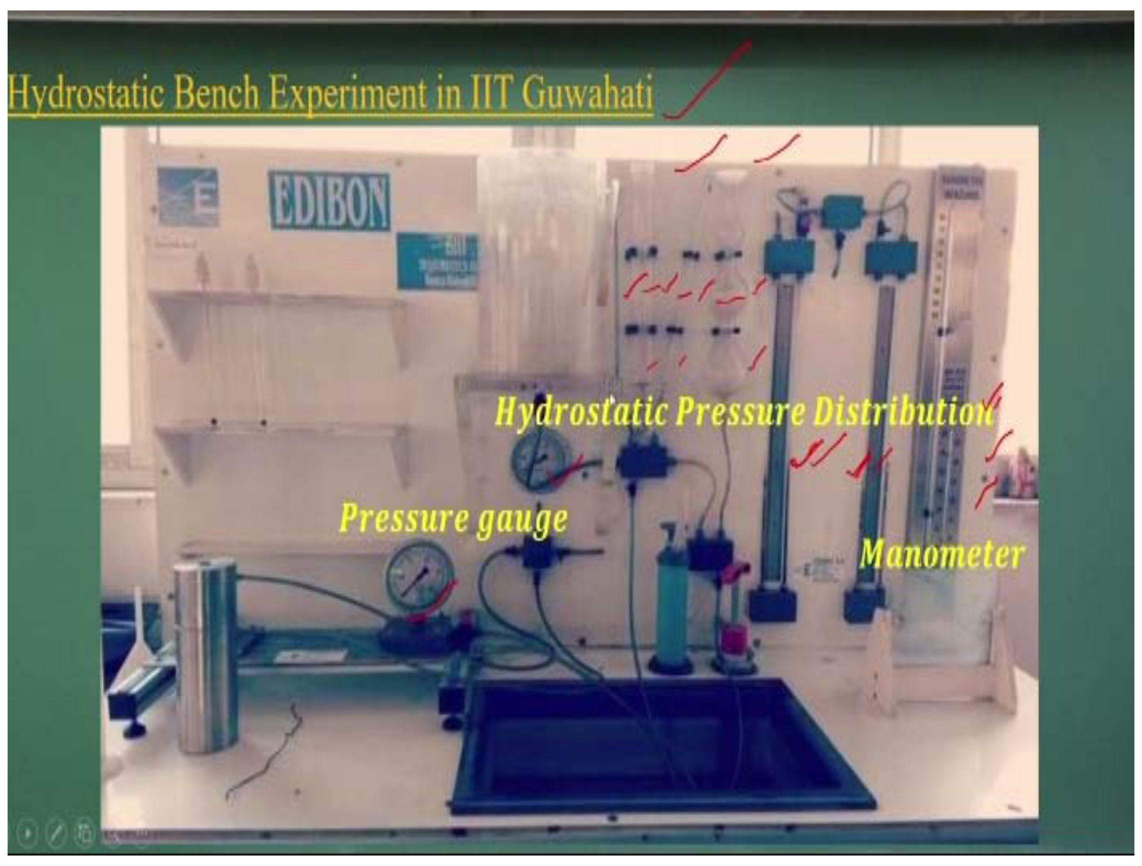

Now let us look at the hydrostatic bench experiment setup, which is there in IIT Guwahati. So this type of experimental setups which is called this hydrostatic bench experiment setup, you can see the pressure gauge, you can see the pressure gauge. You can see this mercury manometers. So these are mercury manometers are there. These are U-tube manometers are there.

And you can have conduct different experiments using this pressure gauge as measurements, the manometer measurement, and U-tube manometers. Not only that, there is the experiment setups to prove the Pascal's laws that the pressure in a horizontal surface remains the constant. That what you can see there is a different shape of the containers are there.

So when you fill up the fluid if it is at the rest you will have the same horizontal plane will be developed irrespective of whatever the shape of the containers. That is the hydrostatic if they are connected it. This is what we discuss more detail while teaching the hydrostatics, basic hydrostatics concepts and we also derive the equations for that.

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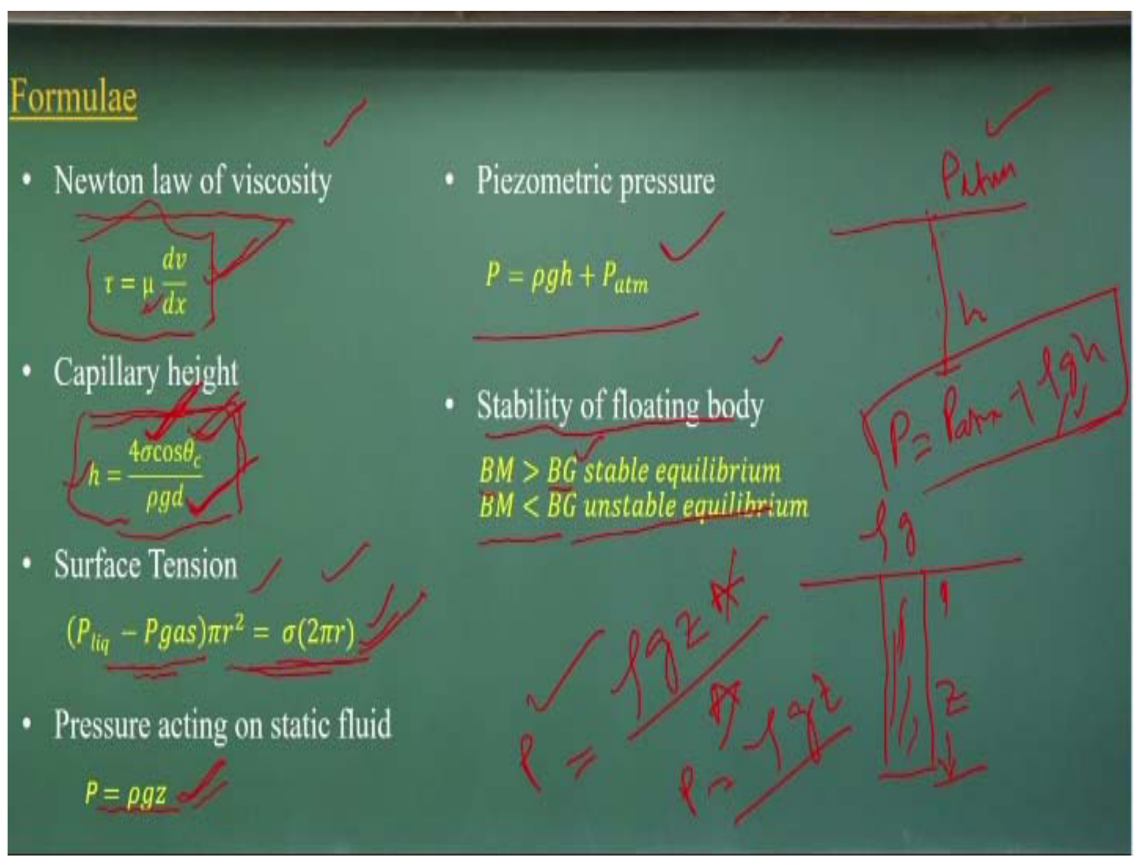

Now let us I just write the formulaes, okay. And just to repeat the things what we learnt it in fluid basic properties and the fluid statics that the Newton's laws of viscosities we established the relationship between shear stress and the velocity gradient.

Newton law of viscosity

$$\tau = \mu \frac{dv}{dx}$$

μ =dynamic viscosity

Similar way also we derived the capillarity height in terms of the diameter of the capillarity tube.

Capillary height

$$h = \frac{4\sigma\cos\theta_c}{\rho g d}$$

θ_c = angle of contact

σ = surface tension force

d= diameter

h=capillary height

The surface tension force when we equate with a pressure difference and the area where it is acting it the force, both the side the force equating what you have done it the force the net force acting on this part is taking care of the force due to the surface tensions.

Surface Tension

$$(Pliq - Pgas)\pi r^2 = \sigma(2\pi r)$$

That what is here and as you know very basic things, when you consider z as a at the free surface level is zero.

As z increases in the downwards the pressure will be

$P = \rho g z$

So that what it indicates of a linear pressure distribution when fluid is at the rest. But, if you consider that the at the atmospheric P equal to P atmospheric pressures and at the height h what will be the pressures which is very simple is

$$P = \rho g h + P_{atm}$$

ρ is the density,

g is acceleration due to gravity.

And next one what we know it, how a floating body's stability we analysis with respect to BM and the BG the distance between the buoyancy to metastatic points, the buoyancy to the center of gravity, that is what will be show us that whether the body is of a floating bodies is a stable or unstable conditions. That what we will get it.

$$BM > BG \; stable \; equilibrium$$

$$BM < BG \; unstable \; equilibrium$$

So let me repeat these things that very simple things as we discuss the Newton laws of viscosity which establish the relationship between shear stress and the velocity gradient. Similar way we derive the capillarity height will be a functions of contact angles, surface tensions, and the d, d stands for here the diameter of the capillarity tube. Or you can equate the force acting due to the pressure difference and the surface tension forces that what we equated.

And very simple, what will be the pressure distribution in a fluid which is at the rest the static fluid will be the weight of the fluid divide by the area.

$$P = \frac{\rho g z A}{A}$$

$$P = \rho g z$$

We will come it or simple way, if I am to know it, the if it is a z is a total depth of a or depth of a fluid and ρ is density g is acceleration due to gravity, the weight will be the $\rho g z A$ divide by the area I get the pressures the weight per unit area, that what I get the pressure.

Simple way P = $\rho g z$. But when you consider P equal to atmospheric then we have considered this part. So very simple way we can compute pressure distributions when fluid is at rest conditions.

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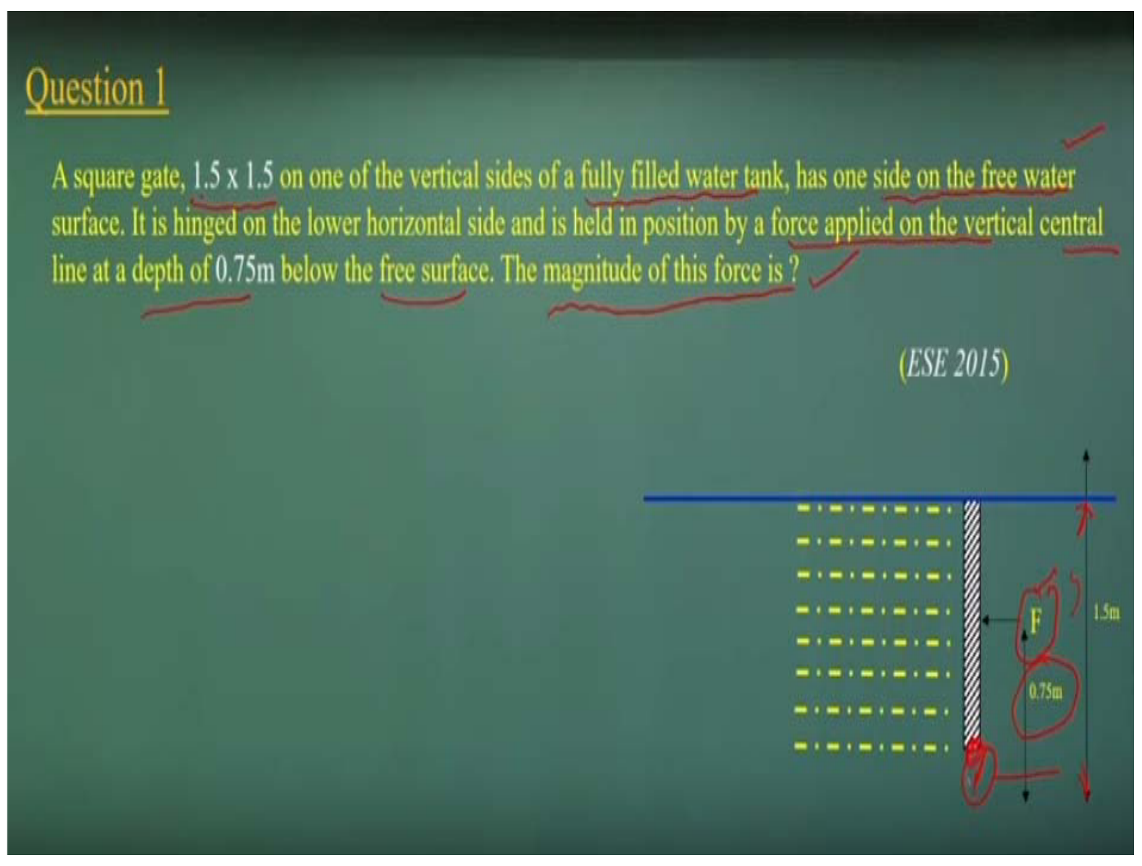

Let us start to solve the problems, these very easy problems that there is a square gate of a dimensions of 1.5 meter into 1.5 meters. One of the vertical sides of a fully filled water tank has one side on the free surface. It is hinged on the lower horizontal sides. Here it is a hinged it and is held in a positions by force applied on the vertical the central line at a depth of 0.75 meter below the free surface.

[A square gate, 1.5 x 1.5 on one of the vertical sides of a fully filled water tank, has one side on the free water surface. It is hinged on the lower horizontal side and is held in position by a force applied on the vertical central line at a depth of 0.75m below the free surface. The magnitude of this force is?]

So this is 1.5 meters and 0.75 meters below means 0.75 meter from the bottom, from the hinge. This is what 0.75 meters, total is 1.5 meters. So what could be the magnitude of this force? What could be this force part. So this is the force. You can easily solve these problems. The problem is that we need to know hydrostatic pressure distributions. Then we can compute the force due to the hydrostatic distribution.

Once I know what is the force is acting because the fluid is at the rest and where it acts the force that the locations. If I know the force, the center of pressures or the force where is acting it, if I know that to the force magnitudes or the at the locations where the force acts then I can take a moment at the hinge locations to compute what will be the force component.

Flow Classification:

Static fluid

Control volume is static

Homogeneous fluid

So the problems wise it is a very easy problems to solve it. Only we need to draw the pressure diagrams, compute the force due to this pressure diagrams and we need to find out where this force acts, the center of pressures, where the force act. Then we try to take the movement at the hinge locations as you know it the sum of the moments should equal to zero if in case of the hinge conditions.

That the conditions we use it to compute the F value. That is very simple things. Let us have F as we used to do the flow classifications. Here is fluid is rest. Need not to draw any control volumes. But if you want to draw a control volumes to draw the pressure diagram for that we can do it but I am not highlighting this the control volume this case.

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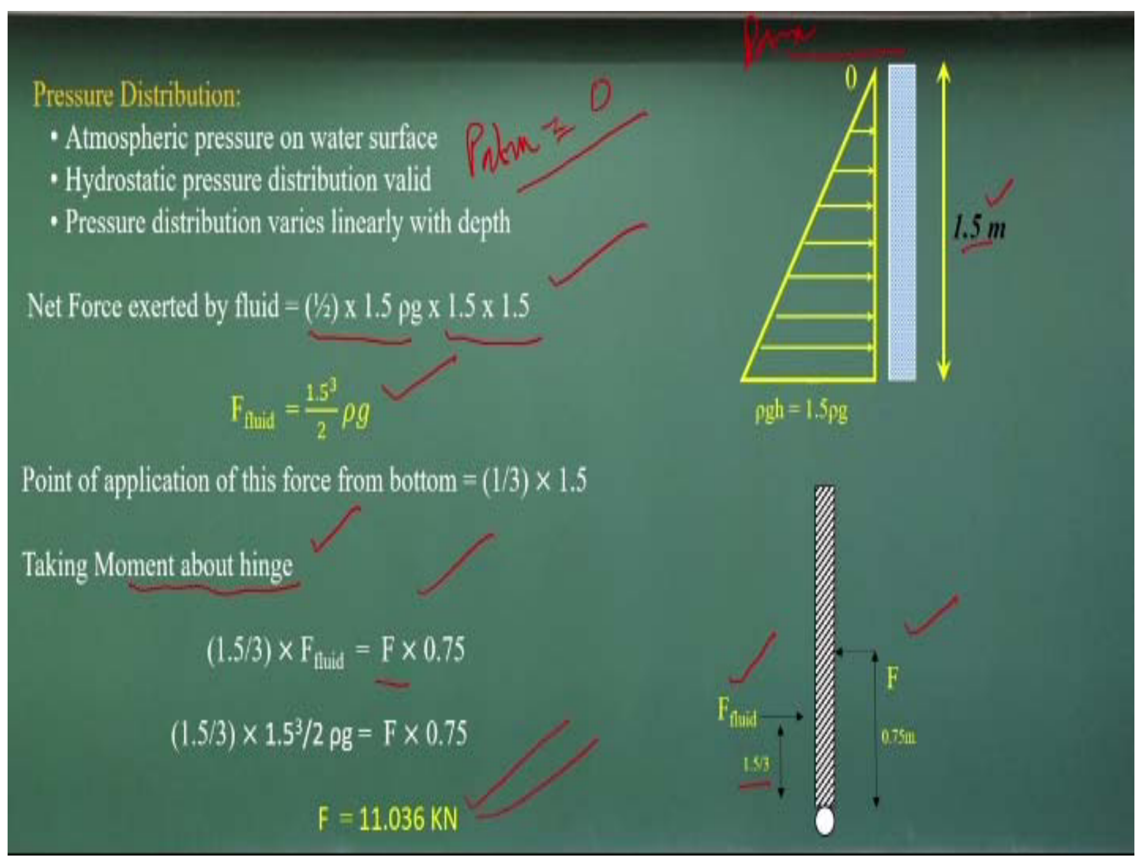

Now the pressures at the water surface levels will be the atmospheric pressures, will be the atmosphere pressures and we can consider this atmospheric pressure is close to the zero okay that as compared to the pressure distributions.

$$P_{atm} = 0$$

This the correct the pressure distribution varies linearly with respect to the depth. That is what will be the pressure distributions.

Net Force exerted by fluid = (½) x 1.5 ρg x 1.5 x 1.5

$F_{fluid} = \frac{1.5^3}{2}\rho g$

The a triangular pressure distributions diagrams we will get it and we need to compute what is the force because of these pressure diagrams. The average pressures multiplied into the area that is what will be the net force due to the fluid at the rest. That what will come it, average pressures into the area of the gate. That is what will be the this is the force will act due to the fluid at the rest and that what will act it at one third distance from the base.

Point of application of this force from bottom = (1/3) × 1.5

Taking Moment about hinge

$(1.5/3) \times F_{fluid} = F \times 0.75$

$(1.5/3) \times 1.5^3/2\ \rho g = F \times 0.75$

$F = 11.036$ KN

As you know in a triangular pressure distribution diagrams the resultant force act at a one third distance from the bottoms or two third distance from the free surface. So one third distance locations that what will be act it the fluid F we know it what is the distance from this. So we just take a moment about the hinge to compute what will be the F, the force into distance, force into distance. That is what is moment we take it the finally it comes out to be 11 KN.

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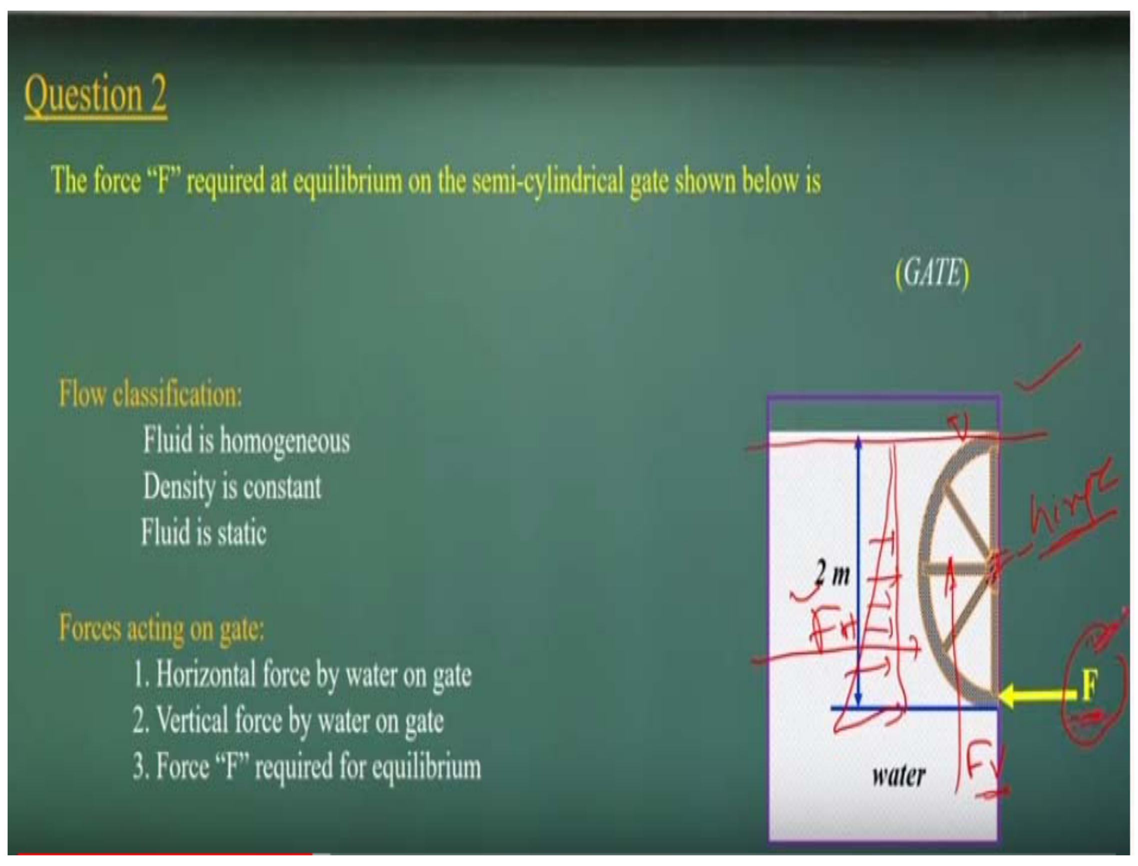

Now let us we another GATE questions we can solve it with as it is a diagrams given here. There is a semi circles cylindrical gate is there and force F is acting here and there is a hinge at this point. So we need to compute it if this is a free surface. This is the two meter depth of the water is there. What could be the force is required if the hinge at this point. This is what the questions.

[The force “F” required at equilibrium on the semi-cylindrical gate shown below is]

Now if you look it, we can have a flow classifications here because fluid is at rest, density is a constant and we can consider is a homogeneous and what we need to compute for this case if you look it this is a curved surface. So because of that there will be horizontal force acting on this. There will be horizontal force act on this. Also the vertical force is going to act on that.

Flow classification:

Fluid is homogeneous

Density is constant

Fluid is static

So there are the two force component will come it. One is a horizontal force and other is vertical force. And another F which is applied to here. So we will try to find out what could be this force component to compute, so that this will not rotate it. So we need to compute what is F is acting on this if there is a two force component, one is a horizontal force because the fluid at the rest.

Forces acting on gate:

1. Horizontal force by water on gate
2. Vertical force by water on gate
3. Force “F” required for equilibrium

Also will be the vertical force F v will act on that. So these force will take a moment about this hinge and that moment taking moment of these three force components will help us to compute what will be the F component. This is what the basic idea to solve these problems.

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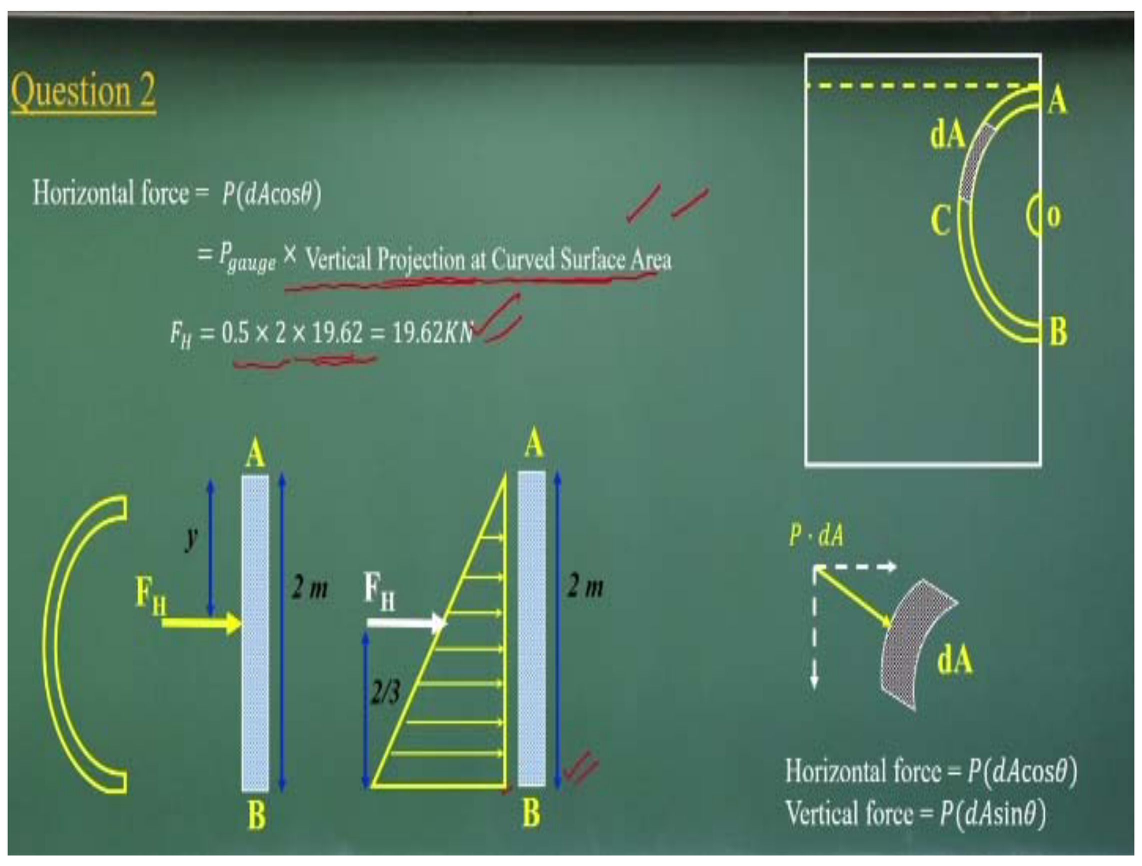

Now to compute horizontal forces, what is going to act it. So it is very easy that the places what will get it that what will be the force into vertical projections of the curved surface area. That is what we derived earlier. So we can compute it what will the force will act on this as the surface the pressure distributions where is like this, we can find out the pressures at the centroid locations.

Horizontal force = $P(dA\cos\theta)$

$= P_{gauge} \times \ Vertical\ Projection\ at\ Curved\ Surface\ Area$

$$F_H = 0.5 \times 2 \times 19.62 = 19.62KN$$

And multiply this area of the vertical projections area of the curved surface that is what is come it which will be the force acting on that. But that force will act to one third distance from this bottoms. That since it is a 2 meter height you have a 2 by 3 meter distance from this B locations. This is what the horizontal force what is acting it.

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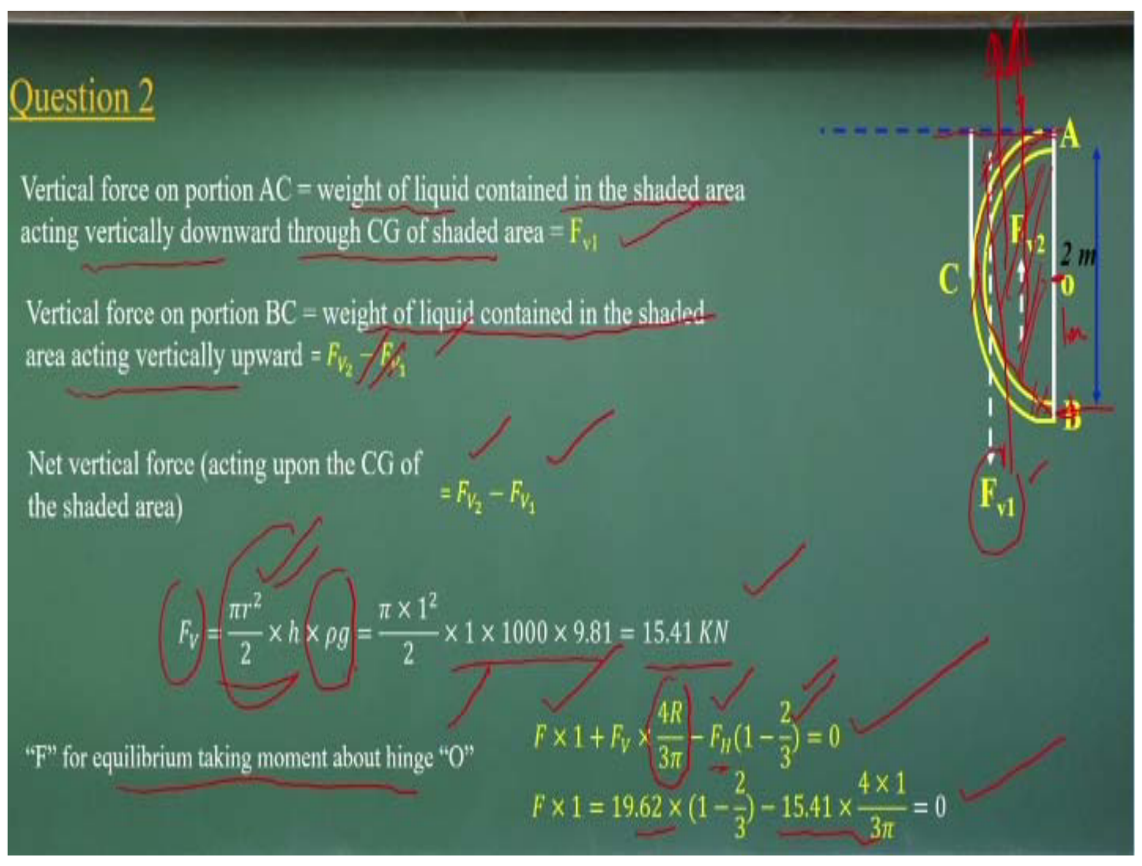

Similar way if I need to compute the how much vertical force is acting on this. So we can find out if this is my free surface. This is the area, the shaded part will give a vertical force component of F_{v1}. But F_{v2} will be the, all the shaded part will be the F_{v2} part. The subtracting this part that what will be the net force what will be act vertically upward directions.

Vertical force on portion BC = weight of liquid contained in the shaded area acting vertically upward $= F_{V_2} - F_{V_1}$

As you know it this F_{V_2} will be the higher than F_{V_1} as the more area of the liquid displaced by this curved surface. So let me compute it. The vertical force on portion of AC that what with the weight of the liquid contained in the shaded area acting vertically downward through CG. That is F_{V_1}. Similar way if you look at what is the force is acting on the BC surface part that what with the weight of the liquid contained in the shaded part okay, which is acting vertically upwards.

Net vertical force (acting upon the CG of the shaded area) $= F_{V_2} - F_{V_1}$

That what will be the F_{V_1} value and the net force will come it the difference between F_{V_2} and F_{V_1}. That is the net force will be acting it. So we can find out the net force will be act it the shaded part of this part only. So that means you just find out the cylindrical area what is the dimension of cylindrical area, volume of the cylindrical area into the specific weight of the water. That is what the ρg.

$$F_V = \frac{\pi r^2}{2} \times h \times \rho g = \frac{\pi \times 1^2}{2} \times 1 \times 1000 \times 9.81 = 15.41\ KN$$

That what will comes out to be 15.41 KN which is the vertical force, the net vertical force acting on this gate parameter. Now if I take a moment at the hinge locations.

"F" for equilibrium taking moment about hinge "O"

$$F \times 1 + F_V \times \frac{4R}{3\pi} - F_H(1 - \frac{2}{3}) = 0$$

$$F \times 1 = 19.62 \times (1 - \frac{2}{3}) - 15.41 \times \frac{4 \times 1}{3\pi} = 0$$

So these are the some of the moment about the hinge should equal to zero. That the concept we have used it. Then we will get it F into 1 that is what will value will come it which becomes zero, the force becomes will be the zero value.

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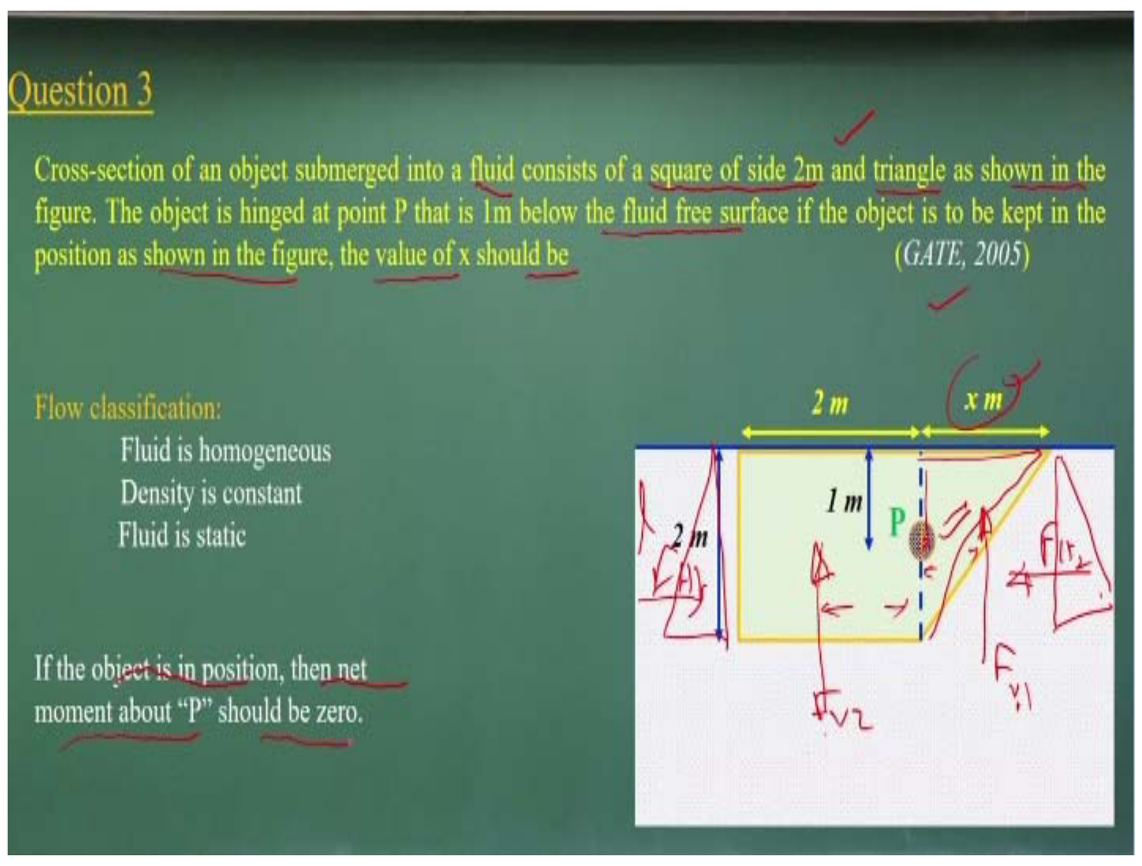

Now let us come to the another questions number 3, the cross sections of object submerged into a fluid consists of a square side of 2 meters and the triangle as shown in the figure. Object is hinged at the point P is 1 meter below the fluid free surface if object is to be kept in this position as shown in the figure, the value of x should be what is the x value so that this will be at a same position, the equilibrium positions.

[Cross-section of an object submerged into a fluid consists of a square of side 2m and triangle as shown in the figure. The object is hinged at point P that is 1m below the fluid free surface if the object is to be kept in the position as shown in the figure, the value of x should be]

This is the hinge locations, the but what is the basic concept of this submerged object. If you look it that there will be horizontal force will be act from these sides. Also horizontal force will go into act from this side. But those will be cancelled out, the same amount of horizontal force will act from this side. Also same amount will be horizontal force FH 1, FH 2 if I consider both will be cancelled out.

What will be the difference? There will be the vertical force will act on this side, also this side F v 1, F v 2 and there is a distance from these locations. So if I know these two forces and their center pressures are the, the line of actions of the vertical force and what is the distance from the hinge. Taking the moment at the hinge, we can compute it what will be the x distance the triangle should be there so that the moment will be the zero.

Flow classification:

Fluid is homogeneous

Density is constant

Fluid is static

That means, the positions will be the same position at the equilibrium positions will be there. Now as object is in a positions that means, the net moment about the P should be zeo, that the concept.

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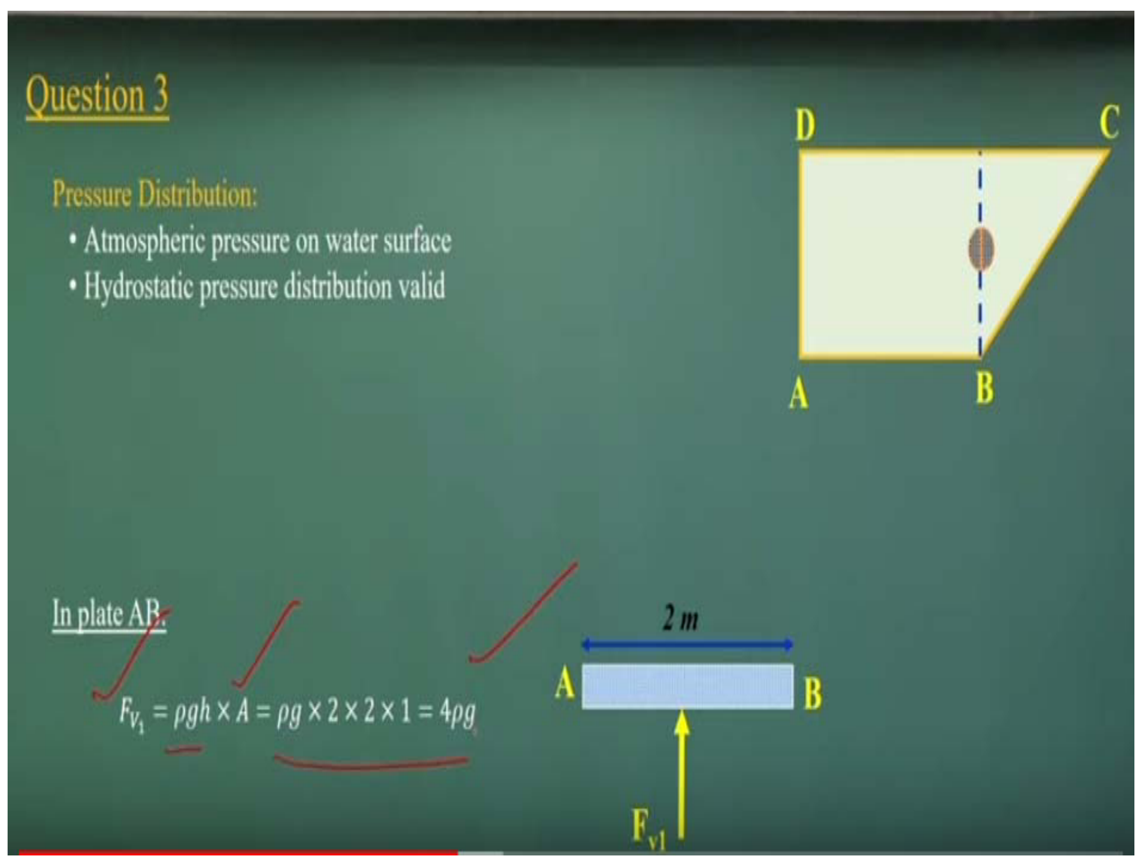

Now let us we compute it what will be the vertical force in the AB part. It is a very simple is the pressure distributions is uniform here and we know rho g into A will be the force component. That what will come it

$F_{V_1} = \rho gh \times A = \rho g \times 2 \times 2 \times 1 = 4\rho g.$

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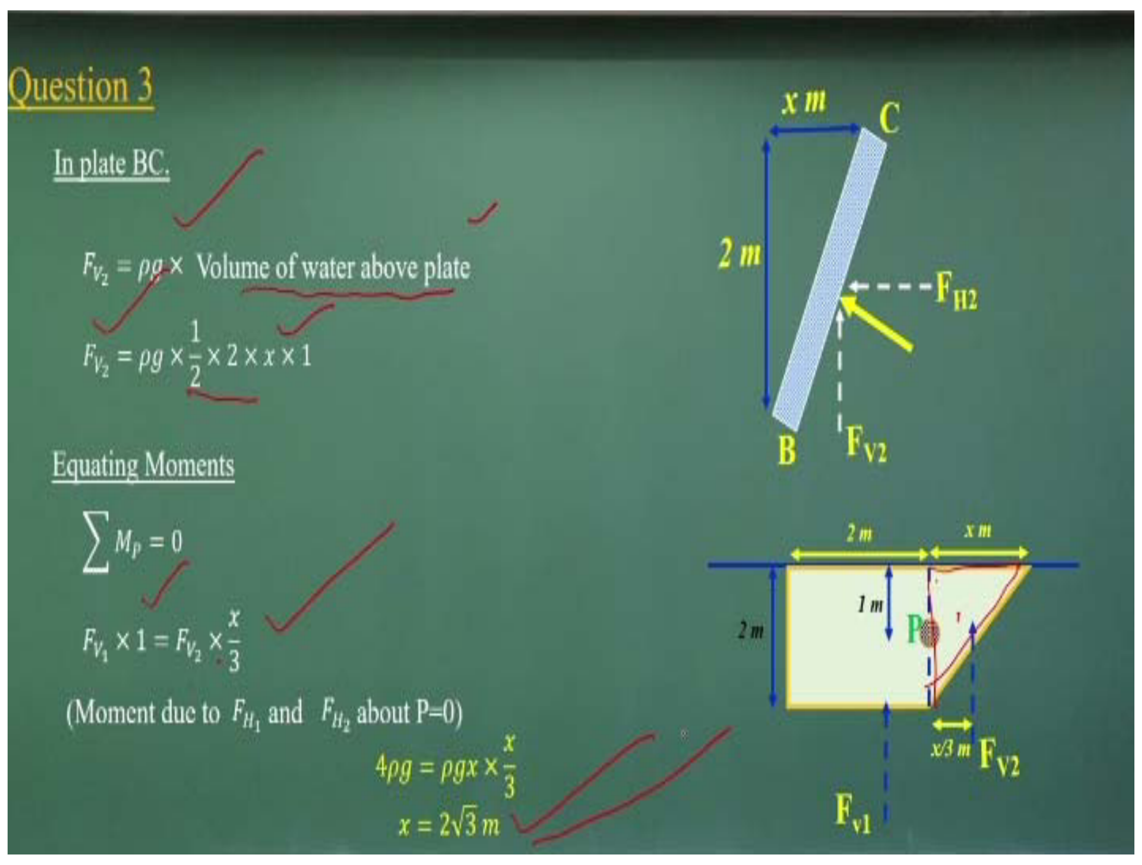

Similar way,

<u>In plate BC</u>

$$F_{V_2} = \rho g \times Volume\ of\ water\ above\ plate$$

$$F_{V_2} = \rho g \times \frac{1}{2} \times 2 \times x \times 1$$

Now we are equating the sum of the moment at the origins that will be,

$$\sum M_P = 0$$

$$F_{V_1} \times 1 = F_{V_2} \times \frac{x}{3}$$

(Moment due to F_{H_1} and F_{H_2} about P=0)

$$4\rho g = \rho g x \times \frac{x}{3}$$

$$= 2\sqrt{3}\, m$$

The CG of this part at this point which is a one by third of the x distance. So that way we can take him a moment at the two locations at the hinge locations and when you get this moment we will get x value of this one. So if you look at these problems, it is very easy problems. Only you have to resolve the force components.

The first you resolve to find out what is the horizontal force is acting it in different surface if this two surface are canceling out, then you look the vertical force components and their line of actions. So if you know the vertical force where is acting it and the vertical line of actions take a moment about the hinge, then you will get the what will be the x value. That is the basic concept here.

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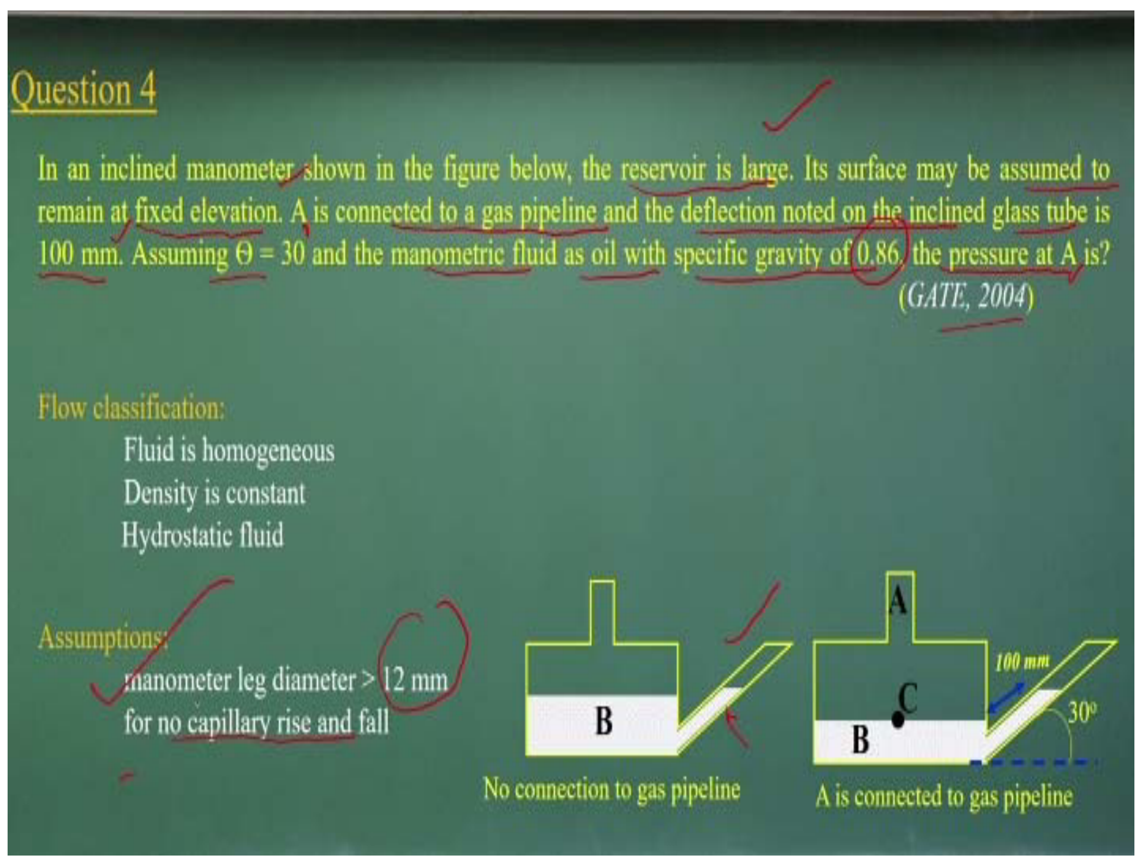

Now let us come it to a manometer problems. So there is a inclined manometer shown in the figure below. Reservoir is large. That is the, its surface maybe assumed to be remain as a fixed elevations. A is connected to a gas pipeline. The deflection is noted on the inclined gas tube. This is what the inclined gas tube is 100 millimeters the theta the angle of inclined manometers theta equal to 30 degrees and manometric fluid as oil with a specific gravity of 0.86.

[In an inclined manometer shown in the figure below, the reservoir is large. Its surface may be assumed to remain at fixed elevation. A is connected to a gas pipeline and the deflection noted on the inclined glass tube is 100 mm. Assuming ϴ = 30 and the manometric fluid as oil with specific gravity of 0.86, the pressure at A is?]

What is the pressure at A point which is a GATE 2004 questions. So that way let we it is a fluid in static problems. So the NCC constants fluid can consider is homogeneous. Now let us have a the assumption is that considering this manometric lake what we have that diameter should be more than 12 millimeters so that there is no capillarity effect.

Flow classification:

Fluid is homogeneous

Density is constant

Hydrostatic fluid

Because of the capillarity there should not be the rise and fall of the manometric liquid for this case. So this is what the assumptions for this problems.

Assumptions:

manometer leg diameter > 12 mm   for no capillary rise and fall

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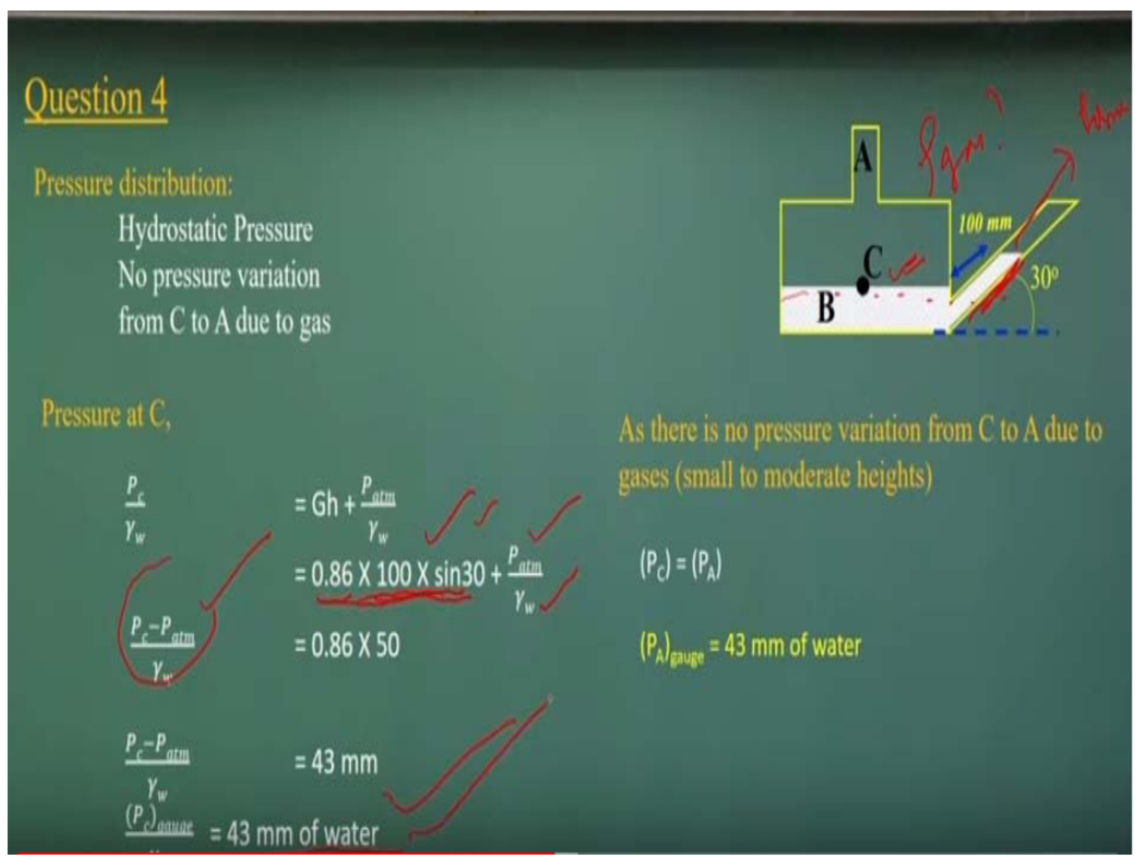

Now what I doing it to equate the pressures to compute what will be the pressures acting on this. So if you look it that there is no pressure variations between the C to A. This is the gas part. Assuming it that since the density of gas is very less. So let us assume it there is not significant difference, variations of the pressure at the P and C levels. If it is that case, the PC the pressure at these locations, we can compute it as at this point the pressure is equal to the atmosphere pressures.

Pressure at C,

$$\frac{Pc}{\gamma w} = \text{Gh} + \frac{Patm}{\gamma w}$$

$$= 0.86 \text{ X } 100 \text{ X } \sin 30 + \frac{Patm}{\gamma w}$$

$$\frac{Pc - Patm}{\gamma w} = 0.86 \text{ X } 50$$

$$\frac{Pc - Patm}{\gamma w} = 43 \text{ mm}$$

$$\frac{(Pc)gauge}{\gamma w} = 43 \text{ mm of water}$$

So the PC is equal to be the height of this manometric liquid, the vertical height that what will come it the 100 X sin30 that will be showed as the vertical height and the specific gravity if I multiplied it, it is talking about in terms of millimeters, in terms of millimeters as equivalent of water how much the pressures is acting on that.

That is what is atmospheric pressures divide by the unit weight of the waters. So that way it is equivalent to water we have done it. So that way the gauge pressure the pressure difference between the PC minus P atmospheric which is the gauge pressure will come out to be 43 mm of the water value.

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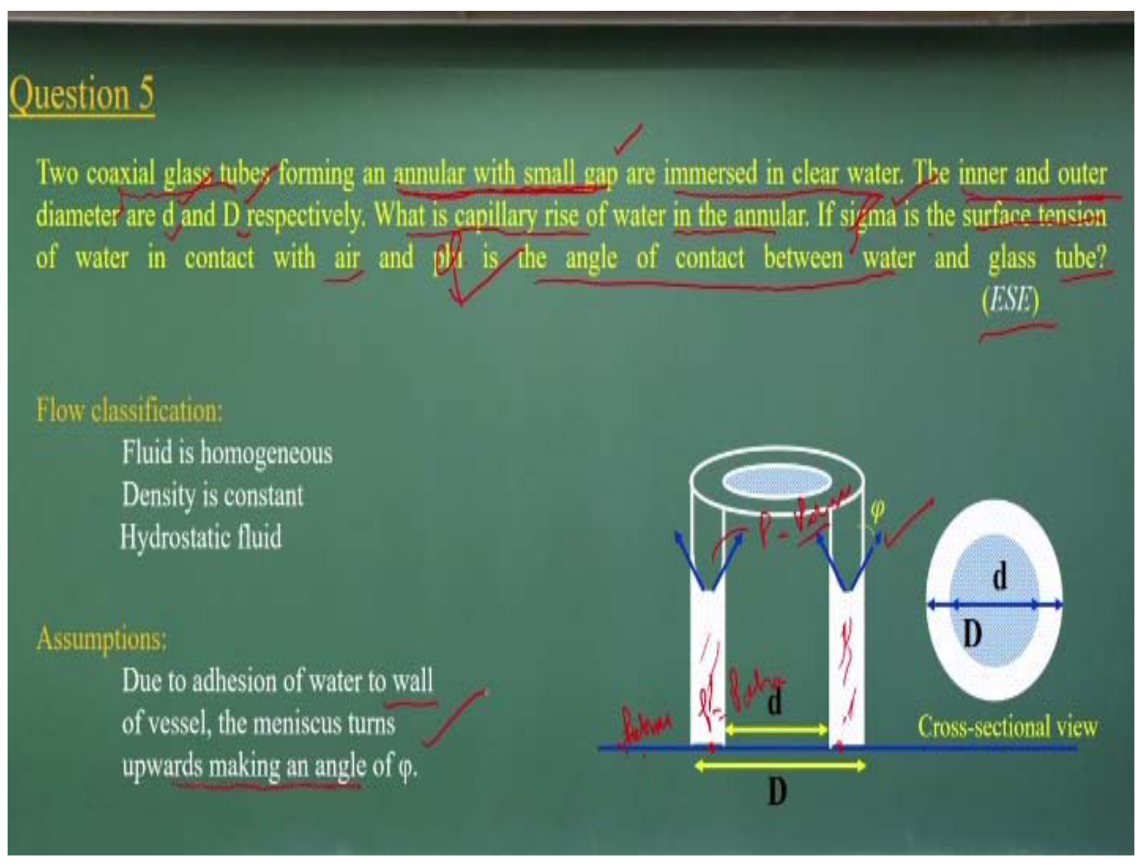

Now let us go to the question number 5 which talking about the capillarity and the surface tensions. So this is what the Engineering Service exam paper where the two coaxial glass tubes forming a annular with a small gap okay. There will be annular but the gaps are small immersed in a clear waters. The inner and the outer diameters are small d and the capital D respectively.

[Two coaxial glass tubes forming an annular with small gap are immersed in clear water. The inner and outer diameter are d and D respectively. What is capillary rise of water in the annular. If sigma is the surface tension of water in contact with air and phi is the angle of contact between water and glass tube?]

What is a capillary rise of water in this annular if the sigma is a surface tension of the water in contact with air and the phi is the angle of contact between the water and the glass tubes. So these are simple derivations to find out a relationship between capillary rise with the surface tensions and the angle of contact for annular with having inner and the outer diameters of smaller d and the capital D.

This is a very easy concept that as you had derived in the theory classes that whenever you put it any small tubes, annular tubes in a liquid, what we get it that the outside you have P equal to atmospheres here also will have the P equal to be the P atmosphere because when fluid is at that the rest any horizontal plane if you take it the pressure should be equal. So that is what this is the horizontal reference plane.

Flow classification:

- Fluid is homogeneous
- Density is constant
- Hydrostatic fluid

The fluid outside since is it at the atmospheric pressures, the fluid which is inside in this capillary tube also will be the atmospheric pressures. So P will be the atmospheric pressures. If is that is the conditions, here also P equal to P atmospheric pressures. That means, the weight of this liquid which is capillary raised from the surface that what will be balanced by the surface tension force as we derived in the theory class.

Assumptions:

Due to adhesion of water to wall of vessel, the meniscus turns upwards making an angle of φ.

This very basic concept, the fluid due to the adhesion of water to wall the that is what is upwards making angle of φ degree.

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